Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

The locus will be an equilateral hyperbola, if the axis of the parabola is perpendicular to an asymptote of the other conic.

The two points at infinity of the locus (Σ) will coincide, if d_1 is parallel to a_2, *i. e.*, if d_1' is an axis of (Σ_1). Hence: *The required locus will be a parabola, if the axis of the given parabola is parallel to one of the axes of the other given conic.*

If the central conic is a circle, the locus is always a parabola.

In the construction above, if for the diameter q_2 is taken the one n_2 passing through C_1, the corresponding diameter n_1 of (Σ_1) is the tangent to (Σ) at this point. The diameter n_1 will coincide with n_2, if the conjugate directions of n_2 in (Σ_1) and (Σ_2), respectively, are perpendicular to each other. The line n_2 is then a part of (Σ). Hence: *If the directions conjugate in the two given conics to the line of their centers, are perpendicular to each other, the locus degenerates into two straight lines, one of which is their line of centers.*

The reader may discuss the case when the axis of the parabola passes through the center of the other given conic.

III. Two parabolas. The required locus passes through the points at infinity of the given parabolas (Σ_1), (Σ_2), the conic (Σ) is therefore always an hyperbola. Draw the tangent to (Σ_2) which is perpendicular to the axis of (Σ_1). The diameter of (Σ_1) which passes through the point of contact, is one of the asymptotes of (Σ). The other asymptote is found in the same way starting with (Σ_1). The hyperbola (Σ) will be equilateral, if the axes of the parabolas are perpendicular. If these axes are parallel, their common point at infinity is the center of the two superposed projective pencils (q_1), (q_2), whose double elements constitute the required locus.

B. POLAR PLANES WITH RESPECT TO QUADRICS.

If P is a point of the required locus, its polar planes π_1, π_2 with respect to the two given quadrics (Σ_1), (Σ_2), meet the plane at infinity in two lines p_1, p_2, which are conjugate with respect to the imaginary circle at infinity (γ), *i. e.*, p_2 passes through the pole M of p_1 with respect to (γ). The poles, with respect to (Σ_1) of all the planes passing through p_1, are on the line m_1 which is the conjugate to p_1 with respect to (Σ_1); the poles, with respect to (Σ_2), of all the planes passing through M, are in the polar plane μ_2 of M with respect to (Σ_2). The point P is therefore the point of intersection of the line m_1 with the plane μ_2. If p varies in the plane at infinity, m_1 describes the bundle of lines (m_1) having for its center the center of (Σ_1), and the two forms (p_1), (m_1) are reciprocal; if the point M varies in the plane at infinity, μ_2 describes a bundle of planes (μ_{21}), having for its center the center of (Σ_2), and the two forms (M) and (μ_2) are reciprocal. The two plane forms (M), (p_1) being reciprocal with respect to (γ), the bundle of lines (m_1) is reciprocal to the bundle of planes (μ_2). Hence: *The locus of the point whose polar planes, with respect to two given quadrics, are perpendicular to each other, is, in general, a quadric passing through the centers of the given surfaces.*

Also solved analytically by FLORENCE P. LEWIS.

480. Proposed by CLIFFORD N. MILLS, Brookings, S. Dakota.

Of equal quadrilaterals on the same base, that which has the least perimeter must have the angles not adjacent to the base equal to each other.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

We shall first prove a different theorem and then show it is equivalent to the above. "Of isoperimetric quadrilaterals on the same base, that which has the greatest area must have the angles not adjacent to the base equal to each other." Call the vertices $ABCD$, the given base being AD. The maximum quadrilateral must have $AB = BC = CD$, since, if the triangle ABC is a maximum, it is isosceles. Again, it must be inscriptible in a circle and hence is an isosceles trapezoid with angle B equal to angle C.

Now suppose we could find another quadrilateral, $AB'C'D$, having the same area as $ABCD$, but a smaller perimeter. It is clear that we can then construct a third quadrilateral, *e. g.*, by moving the vertex B' to B'', so that $AB''C'D$ has a larger area than $AB'C'D$ or $ABCD$, but the same perimeter as $ABCD$. But this is impossible. Also, there can be no other quadrilateral having the same area and perimeter. Hence, $ABCD$ has the least perimeter, for the given area, as well as the most area for a given perimeter.